

Binomial Expansion * Recognizing Conic Sections

$$(F + S)^n = F^n + \binom{n}{1} F^{n-1} S^1 + \binom{n}{2} F^{n-2} S^2 + \binom{n}{3} F^{n-3} S^3 + \dots + S^n$$

Ex A: Write the expansion of $(x + y)^5$

$$x^5 + \frac{5!}{4!1!} x^4 y^1 + \frac{5!}{3!2!} x^3 y^2 + \frac{5!}{2!3!} x^2 y^3 + \frac{5!}{1!4!} x^1 y^4 + y^5$$

$$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Ex B: Find the eighth term of the expansion of $(F + S)^{12}$

$$\frac{12!}{(5!7!)} F^5 S^7 = 792 F^5 S^7$$

Ex C: Find the tenth term of the expansion of $(2x^3 - y)^{15}$

$$\frac{15!}{6!9!} F^6 S^9 \rightarrow \frac{15!}{6!9!} (2x^3)^6 (-y)^9 = \frac{5005 \cdot 64x^{18} (-y)^9}{-320320x^{18}y^9}$$

Recognizing Conics

- Circle: x^2 and y^2 separated by + and have the same coefficient
- Ellipse: x^2 and y^2 separated by + and have different coefficients
- Hyperbola: x^2 and y^2 are separated by -
- Parabola: will have x^2 or y^2 but not both

$$\begin{array}{l} x^2 + y^2 \\ x^2 + 4y^2 \\ x^2 - y^2 \end{array}$$

$$\begin{array}{l} y = x^2 \\ x = y^2 \end{array}$$

Ex D: Identify each conic.

- a. $4x^2 + 36y^2 + 40x - 288y + 532 = 0 \rightarrow$ Ellipse
- b. $9x^2 - 54x - 16y - 79 - 4y^2 = 0 \rightarrow$ Hyperbola
- c. $x - y^2 + 4y - 3 = 0 \rightarrow$ Parabola
- d. $x^2 + y^2 - 8x + 6y - 56 = 0 \rightarrow$ Circle
- e. $x^2 + 36y^2 + 40x - 288y + 532 = 0 \rightarrow$ Ellipse